Chapter 11

XENON-135

Build up to Equilibrium
Bulk Transients
Oscillations
Other Fission Products

Why is Xenon a Problem?

- 6.6% of all U-235 fissions produce mass 135 fission products (mainly Iodine 135)
  - Xe-135 is one of the mass 135 fission products
  - Iodine -135 decay to make Xenon-135
- Xe-135 has a large absorption cross section for thermal neutrons.
- At a steady power level, the number of fissions per second is constant, so
  - there is a steady production of I-135
  - Once I-135 has built up to equilibrium, it decays at a steady rate

What Happens on a Power Change?

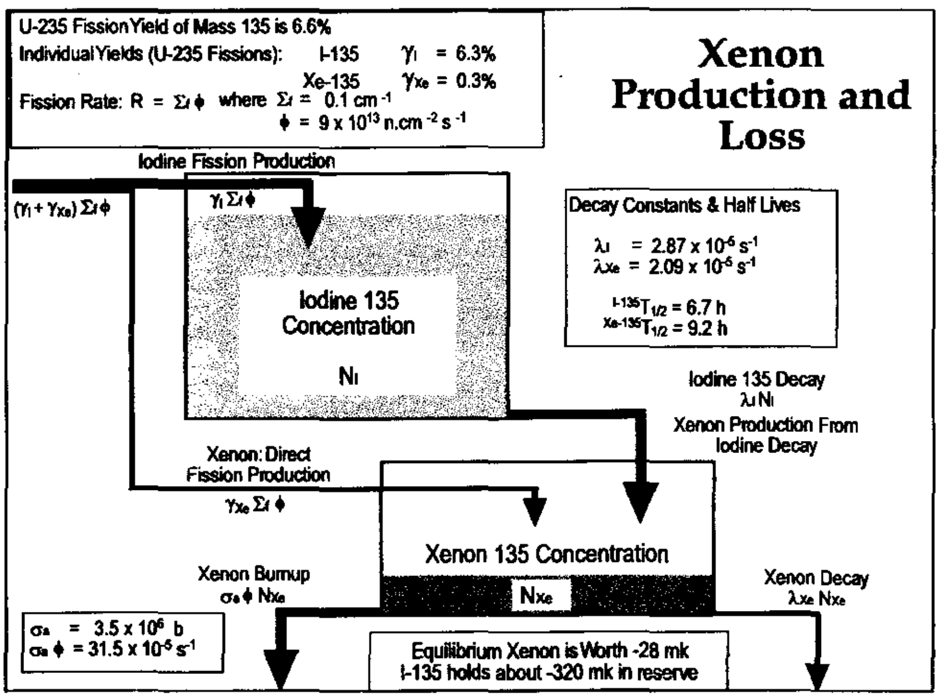
- The high Σ_a for neutrons means xenon burnout changes a lot when flux changes. [$R_a = \Sigma_a \phi$]
- When power increases, the rate of burnout of Xe-135 increases faster than the steady I-135 decay can replenish it.
  - Xenon concentration drops, core reactivity increases
- When power decreases, steady I-135 decay produces more Xe-135 than can be burned out in the lower flux.
  - Xenon concentration increases, core reactivity drops

Won't the I-135 Concentration Change too?

- Yes.
  - Higher fission rate increases the production rate of I-135; lower fission rate decreases it.
- But the build-up half time, $T_{½}^{135} = 6.7$ hrs causes it to take many hours to change
  - Decay and buildup are both governed by $T_{½}$
- Xenon burnout rate changes immediately
  - with burnout half time measured in fractions of an hour at high power

A few extra details.

- There are a few extra complications to consider
  - there is some direct fission production of Xe
    - production is about 5% from direct production 95% from I-135 decay
  - Xe decays in addition to burnout
    - at high flux, over 90% of Xe removal is by burnout
- All of these effects and the equations for production and removal can be summarized on a simple “water tank” flow diagram.



The Tank Diagram

- The tank diagram shows an "analogue computer" for calculating the quantities of xenon-135 and iodine-135
- It can be used to derive differential equations for the Xe and I concentrations
- It can also be used directly as the basis for a numerical computation
- We will use it to derive a variety of quantities that characterize the buildup and transient positive feedback from xenon

Steady Conditions

- Notice that the tank levels remain steady as long as the inflow exactly matches the outflow
- Notice that the two arrows representing decay
  - I decay (= Xe production) and Xe decay

  are not flux dependant
- If the reactor trips, valves shut off flow in all the lines to or from the tanks except the decay lines.

TABLE 1 PARAMETERS

- **For Reference**
- λ_I = 2.93 10^{-5} s^{-1} (G.E. Nuclear Chart 1996)
- λ_{Xe} = 2.11 10^{-5} s^{-1} (G.E. Nuclear Chart 1996)
- σ_a^{Xe} = 3.5 x 10^6 b = 3.5 10^{-18} cm2 (New Transent value is 3.1 10^{-18} cm2)
- γ_I = 6.3% (New Transent value ≈ 6.4 % for equilibrium fuel & 6.3% for U-235 fissions.)
- γ_{Xe} = 0.3% (New Transent value ≈ 0.6 % for equilibrium fuel & 0.24% for U-235 fissions.)
- Σ_f 0.1 cm^{-1} (fresh CANDU fuel) $\Sigma_f \approx 0.089$ cm^{-1} (equilibrium fuelling) is burnup dependent
- $\phi_{F.P.}$ = 9.1 × 10^{13} n cm^{-2} s^{-1} (fuel flux at full power/equilibrium fuelling: BNGSB Xe predictor)
- $\phi_{F.P.}$ = 1.0 × 10^{14} n cm^{-2} s^{-1} is a convenient value for calculation, and close enough.
- time constants for ϕ_{final} = full power flux (for equivalent half lives multiply by *ln*2 = 0.693):
- $\left(\sigma_a^{Xe}\phi_{final} + \lambda_{Xe}\right)^{-1} \approx 49.1$ minutes (half time 34 minutes)
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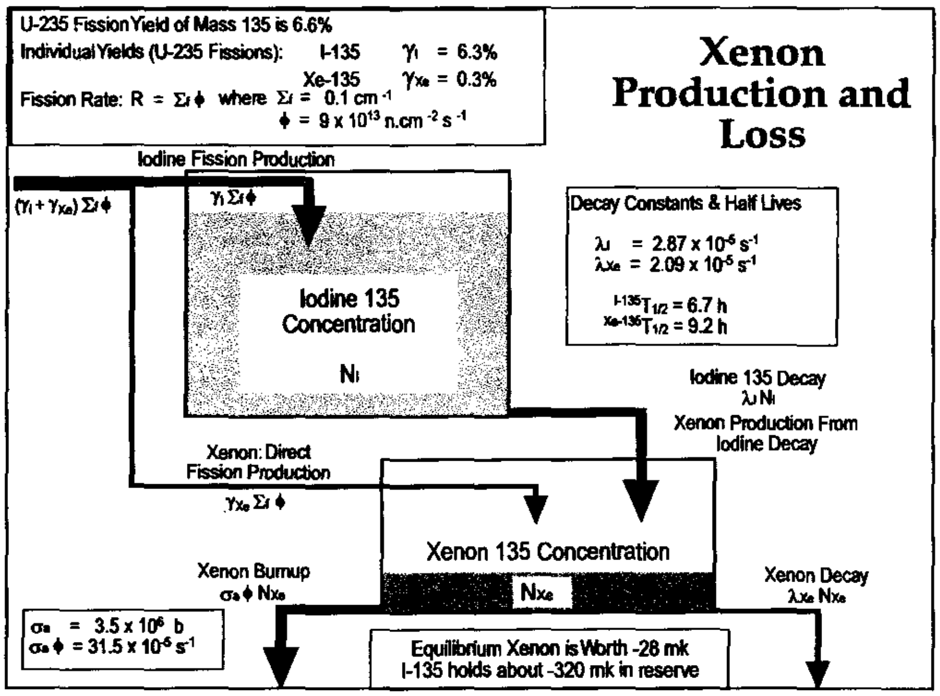
- $\left[\sigma_a^{Xe}\phi_{final} - \left(\lambda_I - \lambda_{Xe}\right)\right]^{-1} \approx 53.7$ minutes (half time 37 minutes)
- $1/\lambda_I$ = 569 minutes (half life 6.6 hours)
- $1/\lambda_{Xe}$ =790 minutes (half life 9.1 hours)
- $1/(\lambda_I-\lambda_{Xe})$ = 2032 min = 33.9 hrs (half time 23.5 hours)
- To convert from number concentration to mk worth of xenon-135, take 1 mk ≈ 6 × 10^{16} atoms

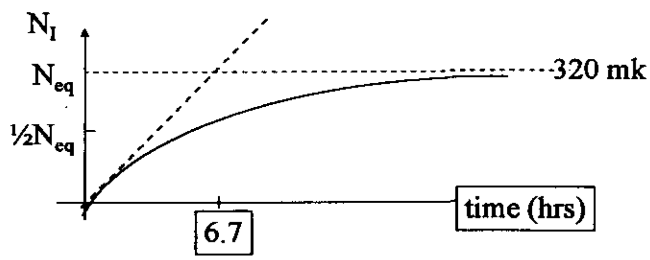


Equilibrium Steady State Conditions for Xenon and Iodine

- *Calculate the fraction of mass 135 fission fragments that are xenon and the fraction that are iodine.*
- *Show that the % of production of xenon once equilibrium is achieved is almost 95% from iodine decay and 5% direct fission production.*
- *Show that the removal of xenon at normal full power flux conditions is more than 90% by burnout and almost 10% by decay.*

Build up to Equilibrium

- At start up there is no xenon or iodine.
- Iodine is produced steadily with constant ϕ
  - a constant production rate gives a steady increase
- But I decays, λN_I, so the more there is the faster you lose it
  - eventually production matches decay, with $N_I = N_{eq}$



The Equation for I Buildup

$$\frac{dN_I}{dt} = \gamma_I \Sigma_f \phi - \lambda_I N_I$$

- Rate = steady production - decay
- equilibrium when production = decay and/or
- rate = 0
- Solution on the Next Slide

Iodine Buildup to Equilibrium

$$N_I(t) = N_{I(eq)}\left(1 - e^{-\lambda_I t}\right)$$

$$N_{I(eq)} = \frac{\gamma_I \Sigma_f \phi}{\lambda_I}$$

- $N_I(eq)$ = - 322 P *mk* is the reserve of iodine waiting (with a half life of 6.7 hours) to become xenon (with parameters from Table)

Equilibrium Iodine

- *Develop formula for equilibrium iodine concentration and show that equilibrium iodine concentration is proportional to steady state flux.*

$$N_{I\ eq} = \gamma_I \Sigma_f \phi / \lambda$$

- *Notice that equilibrium iodine is proportional to flux (neutron power level)*
  - if the reactor operates at 60% F.P. iodine builds to about 0.6 of 322 mk

Xenon Differential Equation

- This one is not so easy: there are 4 terms
  - We will save it and calculate equilibrium Xe first.
- Xe cannot build to equilibrium till Iodine does
- The delay in starting to build until there is significant iodine is called HOLDUP
- Once I is in place, the production is (mainly) at the same steady rate as I production
- equilibrium is reached when production (2 terms) = decay (2 terms)

Equilibrium Xenon

- ***Develop formula for equilibrium xenon concentration and show that the <u>Xenon Load</u> at equilibrium is nearly flux independent for a high flux reactor***
  - Equate the two inflow terms in the xenon tank to the two outflow terms to get the text equation.

$$N_{Xe(eq)} = \frac{(\gamma_I + \gamma_{xe})}{\lambda_{xe} + \sigma_a^{xe}\phi}\Sigma_f\phi = \frac{(\gamma_I + \gamma_{xe})}{\sigma_a^{xe}\phi\left(1 + \frac{\lambda_{xe}}{\sigma_a^{xe}\phi}\right)}\Sigma_f\phi = \frac{(\gamma_I + \gamma_{xe})}{\left(1 + \frac{\lambda_{xe}}{\sigma_a^{xe}\phi}\right)}\frac{\Sigma_f}{\sigma_a^{xe}}$$

Equilibrium Xenon Concentration

- ***For P = 0.6 (60% F.P) equilibrium xenon is only a few mk from its full power equilibrium value***

$$N_{Xe(eq)} = \frac{(\gamma_I + \gamma_{Xe})\Sigma_f}{\sigma_a^{Xe}\left(1 + \frac{\lambda_{Xe}}{\sigma_a^{Xe}\phi}\right)}$$

  - 28 mk for $N_{Xe(eq)}$ and 0.9×10^{14} specify the particular reactor. Other values are physical constants

$$N_{Xe(eq)} = \frac{-28\text{mk} \times P}{0.94P + 0.06} = \frac{-28\text{mk}}{\left(0.94 + \frac{0.06}{P}\right)}$$

Determine the relative concentrations of iodine and xenon and use equilibrium xenon mk worth = 28 mk absorption to calculate the reserve of xenon stored as iodine (Iodine Load).

$$\frac{N_{I(eq)}}{N_{Xe(eq)}} = \frac{\gamma_I}{(\gamma_I + \gamma_{xe})} \frac{(\lambda_{xe} + \sigma_a^{xe}\phi)}{\lambda_I}$$

- $= 0.95 \times (2.11 + 35)/2.93) = 12$
  - so with equilibrium xenon 28 mk, these values give equilibrium iodine = 336 mk
    - cf. 320 mk on an earlier slide, and in the tank diagram

HOLDUP -The Complicated Time Dependence of Xenon Buildup.

- The messy second term only changes things a little bit at the beginning of the buildup

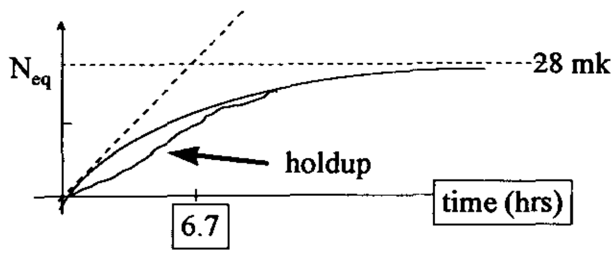
$$N_{Xe}(t) = N_{Xe(eq)}\left(1 - e^{-\lambda_I t}\right)$$ ← NOT A TYPO

$$- N_{Xe(eq)}\left[\frac{\lambda_I}{\sigma_a^{Xe}\phi - (\lambda_I - \lambda_{Xe})} \cdot \frac{N_{I(eq)}}{N_{Xe(eq)}} - 1\right]$$

$$\times \left(e^{-\lambda_I t}\right)\left(1 - e^{-\left[\sigma_a^{Xe}\phi - (\lambda_I - \lambda_{Xe})\right]t}\right)$$

Diagramatically

- Check
  - for t = 0 both terms have a factor $(1 - e^{-ct}) = 0$
    - so at t = 0 everything is zero
  - as $t \to 0$, term 1 has an $e^{-ct} \to 0$ and term 2 $\to 1$
    - $N \to N_{eq}$ = 28 mk (instead of 320 mk for I)
- term 1 builds up like I, with the same $T_{1/2}$
- term 2 starts at 0 and is 0 again in a few $T_{1/2}$



Trip from Equilibrium Steady State

Large Xenon Transient Increase

Xenon After a Trip from Equilibrium Steady State

$$N_{Xe}(t) = N_{Xe(eq)} e^{-\lambda_{Xe} t}$$

$$+ \frac{\lambda_I}{\lambda_I - \lambda_{Xe}} \cdot N_{I(eq)} \cdot \left\{ e^{-\lambda_{Xe} t} - e^{-\lambda_I t} \right\}$$

- Its simple in numbers

$$Xe(t) = 28 e^{-\lambda_{Xe} t}$$

$$+ 3.6 \times 322 \cdot \left\{ e^{-\lambda_{Xe} t} - e^{-\lambda_I t} \right\}$$

Description

- The first term is just the decay of the 28 mk present at the time of the trip
- The second term accounts for the fact that every Iodine that existed at the moment of the trip (about 322 mk worth) must go through both decays
  - when the iodine decays, reactivity drops
  - when the xenon decays, reactivity recovers
- The peak develops because the iodine decay rate is bigger than the xenon decay rate
  - so the difference leaves a large + transient.

Time to the Peak

- It is straightforward, but not necessarily easy, to take a time derivative of the xenon transient equation and set the result to zero
- zero slope implies that at some time after the transient starts, with Xe increasing and I decreasing, the production and decay of Xe will be equal
- This is the peak, and the equation can be solved for time to the peak.

Time to the Peak of the Transient

(trip from equilibrium steady state)

$$t_{peak} = \frac{1}{\lambda_I - \lambda_{Xe}} \ln\left[\frac{\lambda_I}{\lambda_{Xe}}\right] - \frac{1}{\lambda_I - \lambda_{Xe}} \ln\left[1 + \frac{(\lambda_I - \lambda_{Xe})}{\lambda_I} \cdot \frac{N_{Xe(eq)}}{N_{I(eq)}}\right]$$

- t_{peak} (hrs) = 11.1 - 33.9 *ln*[1 + 0.024/(0.94 P + 0.06)]
  - = 10.3 hours for P = 1 (trip from full power)

Estimating the Size of the Peak

- At the peak, xenon decay = iodine decay, so
- $\lambda_I (N_I^{eq} e^{-\lambda t}) = \lambda_{Xe} N_{Xe}^{peak}$,

$$N_{Xe}^{peak} = \frac{\lambda_I}{\lambda_{Xe}} \cdot \left(\frac{\gamma_I \Sigma_f \phi}{\lambda_I} \right) e^{-\lambda_I t_{peak}}$$

- $N_{xe}^{peak} = (\lambda_I/\lambda_{Xe})$ *322 mk* $2^{-(10.3/6.6)}$ *(trip from F.P)*
  - The estimate of the xenon peak size from this gives N_{Xe}^{peak} near 150 mk.

Some Practical Considerations

- *poison override time* and
- *decision and action time*

- Initial rate of xenon production after the trip is $\lambda_I \times$ (322 mk) - $\lambda_{Xe} \times$ (28) mk = 8.66×10^{-3} mk/s = 0.5 mk/min
- Adjusters, pulled out of core to override xenon, add + 15 to + 18 mk
  - in 30 to 36 minutes the xenon level is too high
  - it probably takes 10 minutes to remove the rods
    - this gives the operator about 20 to 25 minutes to decide

Poison Out Time

- Analysis of the causes of the trip takes more than the decision and action time
  - not in the old days though
- The reactor poisons out
- It takes 35 to 40 hours (for a trip from full power) for the transient to pass and xenon to drop into the range where adjuster removal could make the reactor critical again
- This is called the ***Poison Out Time***

Poison Prevent

- If reactor power drops to the 50% to 60% range from full power the size of the transient is much less
- Small enough, in fact, that the reactor can be kept at 60% power throughout the transient.
  - As Xe builds, zone levels drop to compensate
  - when zones run out of room, RRS drives out a bank of adjusters - zones rise again
  - The process repeats until all adjuster banks are out
  - now xenon starts dropping and the process reverses

Smaller Transients

- Any power change at high power results in a transient
- The size of transient is smaller the smaller the power change
  - the smaller the steady state Iodine difference
- The time to the peak is less for smaller transients.
- On a power rise, xenon *decreases* transiently

Xenon Oscillations

Requirements:
High Flux
Large Size

High Flux and Large Size

- For a noticeable xenon transient to occur, the removal of xenon by burnout must be significantly higher than the removal by decay
  - for CANDU this is somewhere near 25% F.P.
  - spatial control is phased in between 15% & 25%
- For a physically large core, what happens in one region has little direct affect on another region
  - size bigger (by × 6 or so) the distance an average neutron takes to slow down and diffuse (≈ 40 cm.)
- CANDU fits both criteria

Oscillations

- A small flux increase in one region,
  - corrected by bulk power control, giving a small decrease elsewhere
- sets off two xenon transients in opposite directions in two regions of the core
- Even a small flux increase causes increased Xe burnout, less absorption, still higher flux etc.
  - a typical positive feedback loop
- Exactly the reverse happens where flux is low
  - buildup of Xe drops flux even lower, more Xe etc.

Time Dependence

- in the increasing Xe region flux drops, iodine production drops, and many hours later the high Xe level cannot be sustained and it starts dropping
  - once it starts dropping, the feedback effect makes it drop even more, driving it down again
- In the decreasing xenon region flux is rising, fission rate increasing and I production going up. Eventually the extra I makes enough Xe to reverse the direction
  - again, positive feedback forces Xe levels up & flux down

Cyclic Behaviour

- Flux flattens out again, with equilibrium Xe everywhere, but
- The iodine concentrations in the two regions are, simultaneous with normal xenon, at extremely different concentrations
  - The region where flux was falling continues to fall
  - The region where flux was rising continues to rise
- Without intervention the cycling will continue,
  - with the amplitude likely increasing
    - small oscillations may damp out in time (several cycles) but larger ones are self sustaining, an may grow.

Liquid Zone Control to the Rescue

- The cycling itself is hard on equipment, with varying thermal expansions and contractions fighting each other at mechanical joints
- The peak fluxes, and peak channel and bundle powers can be unacceptably high
  - Which explains why instruments are distributed in core to measure differences between zones
  - and reactivity devices (the liquid zones) are distributed in core to offset these differences before they get out of hand

Oscillations in Practice

- Oscillations can be triggered by power changes, fuelling, moderator T changes etc.
- The liquid zone control system should prevent oscillations, or damp them out fairly quickly when they do happen
- However, the regulating system phases out spatial control on either low or high level
  - reserving reactivity for bulk power control
- If a large oscillation develops with the zones near limiting, it may not be possible to the zones to limit it.

Other Fission Products

Promethium-149/Samarium-149

Other Absorbers

- The core contains hundreds of fission products
  - produced in various abundances and
  - with varying neutron absorbing cross sections
- Xe-135/I-135 are by far the most important
- Next in importance are Pm-149/Sm-149
  - Others are:
    - Pm-151/Sm-151
    - Ruthenium-105/Rhodium-105
- Also important, though not fission products,
  - Neptunium-239/Plutonium-239
  - On shutdown, Np-239 keeps making fissile Pu-239
    - significantly increasing core reactivity

Buildup of Promethium, the Precursor to Samarium

- The equation for promethium is exactly the same as the equation for iodine
  - the symbols and numerical values are different

$$N_{Pm}(t) = N_{Pm(eq)}\left(1 - e^{-\lambda^{Pm} t}\right)$$

$$N_{Pm(eq)} = \frac{\gamma_{Pm}\Sigma_f \phi}{\lambda^{Pm}}$$

Samarium Parameters

- For Reference
- $\lambda_I \rightarrow \lambda_{Pm} = 3.63 \times 10^{-6}\ s^{-1}$ (G.E. Nuclear Chart 1989)
- $\lambda_{Xe} \rightarrow \lambda_{Sm} = 0$
- $\sigma_a^{Xe} \rightarrow \sigma_a^{Sm} = 4.2 \times 10^4\ b = 4.2 \times 10^{-20}\ cm^2$ (New Transent value is 4.38 10-20 cm2)
- $\gamma_I \rightarrow \gamma_{Pm} = 1.13\%$ (Nuc. Theory notes)
- $\gamma_{Xe} \rightarrow \gamma_{Sm} = 0$
- Σ_f 0.1 cm-1 (fresh CANDU fuel)
- Σ_f 0.089 cm-1 (equilibrium fuelling) is burnup dependent
- $\phi_{F.P.} = 9.1 \times 10^{13}\ n\ cm^{-2}\ s^{-1}$ (fuel flux at full power/equilibrium fuelling: BNGSB Xe predictor)
- time constants for ϕ_{final} = full power flux (for equivalent half lives multiply by $ln2 = 0.693$):
- $\left(\sigma_a^{Sm}\phi_{final}\right)^{-1} \approx 72.6hrs = 3.0days$ (half time 2.1 days)
- $\left[\sigma_a^{Sm}\phi_{final} - \lambda_{Pm}\right]^{-1} \approx 1375hours = 57.3days$ (half time 39.7 days)
- $1/\lambda_{Pm} = 76hours = 3.2\ days$ (half life 53 hours or 2.2 days)

Samarium Equations

- Its relatively easy to write down the equations by analogy with the I/Xe equations
- Its simpler because Samarium-149 is stable
  - the decay terms are zero
- The difference in parameters produces some surprising differences.

Samarium Buildup to Equilibrium

$$Sm^{149}(t) = \frac{\gamma_{Pm}\Sigma_f}{\sigma^{Sm}}\left\{\left(1 - e^{-\sigma^{Sm}\phi t}\right) - \frac{\left(\frac{\sigma^{Sm}\phi}{\lambda^{Pm}}\right)}{\left(1 - \frac{\sigma^{Sm}\phi}{\lambda^{Pm}}\right)}\left(e^{-\sigma^{Sm}\phi t} - e^{-\lambda^{Pm}t}\right)\right\}$$

$$Sm^{149}(t) = \frac{\gamma_{Pm}\Sigma_f}{\sigma^{Sm}}\left(1 - e^{-\sigma^{Sm}\phi t}\right)$$

- very low flux
  $T_{1/2} = 2.1$ days

$$Sm^{149}(t) = \frac{\gamma_{Pm}\Sigma_f}{\sigma^{Sm}}\left(1 - e^{-\lambda^{Pm}t}\right)$$

- very high flux
  $T_{1/2} = 3.2$ days

$$Sm^{149}(t) = \frac{\gamma_{Pm}\Sigma_f}{\sigma^{Sm}}\left\{1 - \left(1 + \lambda^{Pm}t\right)e^{-\lambda^{Pm}t}\right\} \qquad \phi = \frac{\lambda_{Pm}}{\sigma_a^{Sm}}$$

Equilibrium Samarium is Not Flux Dependent (AT ALL)

$$\lambda_{Pm} N_{Pm(eq)} = \gamma_{Pm} \Sigma_f \phi = N_{Sm(eq)} \sigma_a^{Sm} \phi$$

- so
- $$N_{Sm(eq)} = \left(\gamma_{Pm}\right) \frac{\Sigma_f}{\sigma_a^{Sm}}$$
-

Calculating the mk worth of Equilibrium Samarium

$$\frac{N_{Sm} \sigma_a^{Sm} \phi}{N_{Xe} \sigma_a^{Xe} \phi} = \frac{\gamma_{Pm}}{\gamma_I + \gamma_{Xe}} \times \left[1 + \lambda_{Xe} / \left(\sigma_a^{Xe} \phi \right) \right]$$

- = (1.13/6.6) × [1 + 2.11×10^{-5}/3.18510^{-4}] = 0.171 × 1.066 = 0.1825
- Full Power Xenon is 28 mk so
- Equilibrium Samarium-149 is 0.1825 × 28 = 5.1 mk, Independent of Power
- But, Time to build up is sensitive to power level

Samarium Buildup after a Trip

$$N_{Sm}(t) = N_{Sm(eq)}\left[1 + \frac{\sigma_a^{Sm}\phi}{\lambda_{Pm}}\left(1 - e^{-\lambda_{Pm}t}\right)\right]$$

$$N_{Sm}^{peak} = N_{Sm(eq)}\left[1 + \frac{\sigma_a^{Sm}\phi}{\lambda_{Pm}}\right]$$

- Samarium doesn't decay, so whatever is held up in the precursor bank adds to the total
  - For $\phi = \frac{\lambda_{Pm}}{\sigma_a^{Sm}}$ the peak is double the equilibrium value of 5.1: i.e. about 10.2 mk

Samarium is Not a Problem

- The time to build after a trip is about 300 hours
- On a restart immediately after a poison out, the amount of Sm is insignificantly different than equilibrium
- Long after a trip there is lots of extra reactivity because Xenon has decayed
- And if that is not enough, decay of Np-239 to Pu-239 adds, with a similar time constant, nearly double the reactivity that Sm removes.